

Influence of the gravitational attraction of terraced landforms to precise levelling results

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Abstract: Precise levelling results are affected by the Earth's gravity field, especially in areas of abrupt changes of landscape, such as terraced landforms. To eliminate the effect of the gravity field, gravimetric corrections need to be used in precise levelling data processing. To find the magnitude of the correction, the measured height differences need to be calculated into geopotential number differences. The geopotential numbers then found for each point can be transformed to conventional (either normal or orthometric) height values that account for the change in the gravity field. To estimate the expected range of the gravity correction within a region of terraced landforms, an experiment was proceeded in Estonia. Gravity data were acquired in 2011 in an area where a levelling section crosses the North Estonian Klint (height difference of 30 m within the levelling section). The cumulative gravimetric correction for the given 300 m long section proved to be 1.2 mm. An approximate relation of 0.2 mm per 1 mGal was found between the gravimetric correction and the gravity value change between stations. Also, it was found that within this particular profile the gravimetric correction amounts to 0.2 mm on average for each 5 m of height difference.

Key words: levelling, gravity field, gravimetry, cliff, Estonia

Introduction

The Earth's gravity field affects precise levelling. Therefore precise levelling results need to be corrected by the gravimetric correction that accounts for changes in the gravity field in the measurement area. The magnitude of the correction value appears to be the most significant in areas of abrupt changes of landscape. The objective of current research is to determine the magnitude of the gravimetric correction to levelling results where the levelling profile crosses a terraced landform.

Terraced landforms are areas with abrupt change in heights within a short distance. They can be found everywhere in the world. The highest coastal cliffs are in Canada, Baffin Island and reach 1370 m from the bottom to the top vertically. Inland cliffs can reach up to 1300 or 4600 meters, depending on the strictness of the definition of a terrace (Wikipedia, 2012). In Europe, some of the better known coastal cliffs are in the United Kingdom and Ireland (see an example on Figure 1), also in France, Italy and elsewhere. In Estonia, terraced landforms may reach up to 56 m in coastal regions.



Figure 1. Cliffs of Mother in Ireland that reach up to 214 m (Travlang Travel Guide, 2011)

The magnitude of the gravity correction will be estimated based on actual gravity data acquired near a terraced landform in Tabasalu, a few kilometres west of Tallinn, the capital of Estonia. Calculations use geopotential numbers to determine the differences between measured geometric heights and “gravimetrically corrected” normal heights.

The gravity data were collected for the author's MSc thesis in geodesy that studied gravity field characteristics over and around terraced landforms. The thesis was defended at Tallinn

University of Technology in 2012. In fact, this paper summarizes the part of the thesis that investigated the effect of the gravitational attraction of terraced landforms on the results of precise levelling.

The paper is structured as follows. At first, the theoretical background of the gravity field's relation to levelling is explained. Following that, data collection and processing methods used in the study will be described. Later, the theoretical methods will be applied to the data gathered; gravimetric correction values calculated and investigated. This leads to a discussion on the matter. A brief summary concludes the paper.

Theoretical background

As is well known the Earth's gravity field is not constant worldwide, but varies according to the locations of observation points. The gravity field is stronger on the poles and weaker on the equator due to the centrifugal effect of rotation of the Earth. The variations in the gravity field are also due to the heterogenous nature of the Earth's interior and the crust. Accordingly, the equipotential surfaces of the Earth are not parallel to each other (Figure 2). Where the gravity field is stronger, the distance between the equipotential surfaces is smaller (for example $g_0 > g_3$, cf. Fig. 2).

Let us look at levelling across such a heterogenous gravity field (cf. Fig. 2). The starting point A is located at the reference surface of levelling (for example on the geoid), which is an equipotential surface W_0 . To find the

height of the endpoint B that is located on the equipotential surface W_B , geometric levelling is conducted. The measured height H' between points A and B is the sum of measured height increments dh' (cf. Fig. 2):

$$H' = \sum dh'_i \quad (1)$$

The actual orthometric height of point B is the length of the plumb line (passing through the point B) in between the reference surface W_0 and the surface W_B . Thus, it is the sum of plumb-line increments dh between the two surfaces (cf. Fig. 2):

$$H = \sum dh_i \quad (2)$$

Since the equipotential surfaces are not parallel i.e. $dh' \neq dh$, the measured height is not equal the orthometric height i.e. $H' \neq H_{AB}$. This creates a situation where the measured height depends on the trajectory of the levelling route, which means that in a closed levelling loop the sum of height increments is usually not equal to zero:

$$\oint dh' \neq 0 \quad (3)$$

To avoid vagueness in the matter, the height increment dh'_i within a section i (see Figure 3) is calculated into geopotential value increment dC_i by multiplying dh_i with the average gravity value g_m on the section:

$$dC_i = g_m \cdot dh'_i \quad (4)$$

where g_m is usually calculated as the average between gravity values of section's endpoints:

$$g_m = \frac{g_j + g_{j+1}}{2} \quad (5)$$

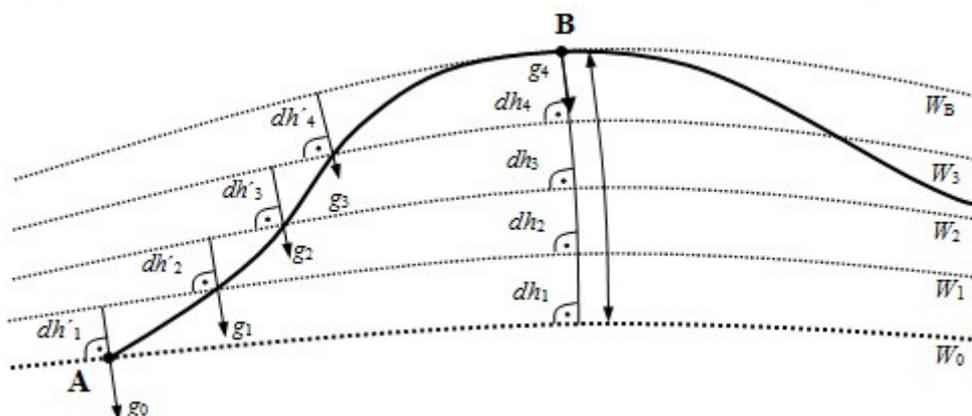


Figure 2. Equipotential surfaces, levelled heights and orthometric heights (Ellmann and Oja, 2008; Fig. 6.5)

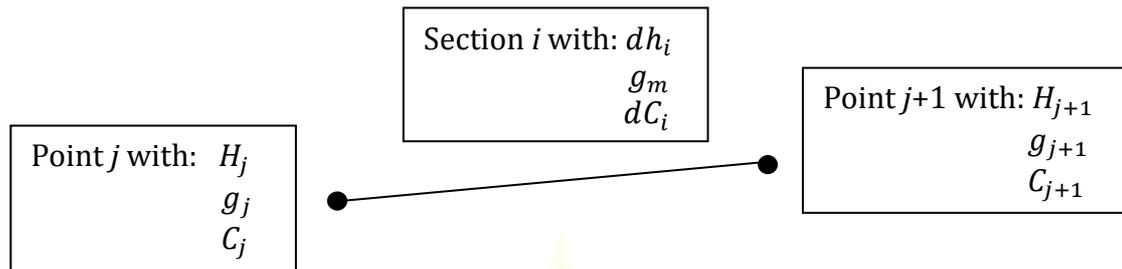


Figure 3. Two levelling stations and a section between them with corresponding values of height, gravity and geopotential

The geopotential value of point B is then calculated from the geopotential value of point A and the sum of geopotential increments dC_i :

$$C_B = C_A + \sum dC_i \quad (6)$$

In a closed levelling loop the theoretical sum of geopotential increments equals to zero and then the remaining deviation from zero reflects only inaccuracies of measurements that can be adjusted by the usual methods (e.g. proportionally to section lengths or by least squares adjustment).

Having found the geopotential value of point B, it is then converted to a conventional height value by:

$$H = \frac{C}{g} \quad (7)$$

The value of g_0 depends on the height system. Since the height system used in Estonia and in many other European countries is the normal height system, the value of g corresponds to $\bar{\gamma}$, the average value of normal gravity along the field line of normal heights (that can be taken to be in the same direction as the normal of the ellipsoid). Therefore, heights are calculated as normal heights by:

$$H^n = \frac{C}{\bar{\gamma}} \quad (8)$$

The value of $\bar{\gamma}$ can be rigorously calculated from (Heiskanen and Moritz, 1967; Eq. 4-42):

$$\bar{\gamma} = \gamma_0 \left[1 - (1 + f + m - 2f \sin^2 \varphi) \frac{H}{a} + \frac{H^2}{a^2} \right] \quad (9)$$

where a is the major semi-axis of the reference ellipsoid; $f = (a - b)/a$ is the flattening of the ellipsoid; b is the minor semi-axis of the ellipsoid; $m = \omega^2 a^2 b / GM$; ω is the angular velocity of the Earth's rotation and GM is the gravitational mass constant.

The geopotential value of the reference point A used in the equation (6) is also calculated from its height H and normal gravity $\bar{\gamma}$ by multiplying the two.

The difference between the measured geometric height H' and the normal height H^n is the cumulative gravimetric correction along the levelling profile. The estimation of the magnitude of the aforementioned gravimetric correction is the main objective of current research. The gravimetric corrections can be calculated either for the full levelling profile or by each individual levelling station.

Data Collection

To obtain precise gravity data for the study of gravimetric corrections, gravity measurements were conducted in Tabasalu, North Estonia (Figure 4). A profile of gravity values was measured on a road that crosses a terrace, the North-Estonian Klint (on the background of Figure 6). The road, being cut into the terrace, is steep, but levelling along it is possible. In fact, a section of the Tallinn levelling network follows the same route.



Figure 4. Location of the study area. Inset – the immediate study area.



Figure 5. LaCoste&Romberg model G gravimeter used for the collection of gravity data

Gravity data were measured using LaCoste&Romberg model G (LCR G-65) relative spring gravimeter (Figure 5). On each point, at least three readings were taken. The reading reflecting the gravity value was always reached by turning the metering screw clockwise. To avoid temperature changes within the instrument, it was kept hidden from direct sunlight (by an umbrella seen on Figure 6) and during transport it was covered with a white cover. The instrument was handled with extreme care as not to jolt or shock it. On one instance the instrument did receive a small shock, the consequences of which were seen in the data and treated as a jump in the drift function. During measurements unnecessary movements around the instrument were avoided, readings were not taken when large trucks passed. Measurements were repeated on several points to allow for drift calculation.



Figure 6. Measuring gravity data on the road that cuts into the North-Estonian Klint (on the background)

The coordinates of the gravity points were determined in the EUREF-EST geodetic system (longitude L , latitude B and height h). Later the coordinates were transformed into Estonian rectangular L-EST'97 coordinates x , y and the Baltic Height System BK'77 heights H . Coordinates were measured by RTK GPS (Real Time Kinematic Global Positioning System), using the Trimble VRS Now virtual stations' service and a Trimble 5800 GPS receiver (also visible on Figure 6). The expected uncertainty of planar coordinates is $\pm 1...2$ cm, of heights ± 3 cm. However, previous experience has revealed in some occasions that the uncertainty can be larger than this, usually within ± 15 cm (Oja et al., 2010).

Data processing

First, the effect of lunar tides was removed from the gravity data by GRAVS2 software developed by the Estonian Land Board (Estonian Land Board, 2012). Next, the gravimeter's drift was modelled by a linear function and removed from the data. A gravimeter's drift is the time-dependent variation of the beginning of its scale that reflects a number of inaccuracies caused by instrumental and environmental factors. To determine the drift values, measurements need to be repeated several times on the same points.

The distribution of drift values calculated by the linear function was random, which proves the function to be suitable (as opposed to

systematically changing values e.g. positive at the beginning of the day and negative at the end).

From the relative differences in gravity values measured between the points and the known gravity value (during the epoch of 2007.5) of 981823.9303 ± 0.0030 mGal (Oja et al., 2009) on Suurupi gravimetric point (situated some 10 km West from the study area), the absolute gravity values on points were calculated. For this, a least square adjustment was proceeded with the data, using the known value of the Suurupi point and known gravity increments between points. The gravity values between Suurupi and points in the measured profile were determined by a quartz spring (digital) Scintrex CG5 gravimeter belonging to the Estonian Land Board. The uncertainty of measurements with the instrument does not exceed the uncertainty of LCR gravimeters.

From the adjustment covariation matrix, standard deviations (STDEV) for the gravity values were obtained. The largest deviation value reached 0.06 mGal. Since calibration parameters determined for the G-65 gravimeter (Oja, Türk and Ellmann, 2010) were not taken into account, the possible loss in accuracy was calculated from the calibration parameters. The largest change in gravity values measured on the profile was 6.6 mGal which corresponds to the calibration parameters' polynomial component (F_{pol}) of 0.011 mGal and periodic component (F_{per}) of 0.010 mGal. The resulting uncertainty of gravity measurements was found to be

$$\begin{aligned}\sigma(g) &= \sqrt{STDEV^2 + F_{pol}^2 + F_{per}^2} \\ &= \sqrt{0.06^2 + 0.011^2 + 0.010^2} \\ &= 0.062 \approx 0.070 \text{ (mGal)}\end{aligned}\quad (10)$$

The free-air gravity correction is about 0.31 mGal/m, which means that the gravity field weakens by 0.31 mGal with every meter that the observation point moves higher from the initial surface. That is because of the gravitational attraction of a body being stronger the closer the observation point. As the uncertainty of GPS height measurements could reach up to 15 cm, the gravity value changes in the worst case scenario for about $-0.31 \cdot 0.15 = -0.05$ mGal.

Hence, the accuracy of the heights obtained from the GPS measurements is sufficient for the purposes of this research.

Alternatively, also existing gravimetric data can be used to determine gravity values on levelling stations' points. As an outcome of the Estonian Science Foundation grant ETF7356 a gravity anomaly database covering Estonia has been compiled. Using interpolation and initial height values, gravity values on levelling points can be found from the gravity anomaly database. These have proved to have the accuracy of about ± 0.6 mGal in terraced areas (Talvik, 2012; Ch. 6.2).

Calculations of the gravimetric correction

As described before, to eliminate the effect of the gravity field change along a levelling profile, the height differences measured need to be converted into geopotential numbers, if necessary, adjusted and later converted to conventional normal height values. This has been done using the gravimetric data collected on the Tabasalu profile, assuming the accuracy of height measurements in Tabasalu to be the same of precise levelling.

The measured geometric height difference between the endpoints A and B of the profile is:

$$\begin{aligned}\Delta H &= H(B) - H(A) = 32.490 - 2.576 \\ &= 29.914 \text{ (m)}\end{aligned}$$

The normal height difference between the endpoints calculated using formulae (4), (5), (6) and (8) to determine the normal heights amounts to:

$$\begin{aligned}\Delta H^n &= H^n(B) - H^n(A) = 32.4887 - 2.5759 \\ &= 29.9128 \text{ (m)}\end{aligned}$$

The discrepancy between the two height differences is therefore:

$$\begin{aligned}\Delta H - \Delta H^n &= 29.9140 - 29.9128 = 0.0012 \text{ (m)} \\ &= 1.2 \text{ (mm)}\end{aligned}$$

Hence, the cumulative gravimetric correction along the Tabasalu profile would amount to 1.2 mm. This is surprisingly large and very significant in terms of precise levelling.

Let us also investigate the distribution of the correction value along the profile. For that, the value of $H - H^n$ for every levelling station is plotted against the distance of the gravity point from the terrace (Figure 7). The distance is calculated as a planar distance from the point that is situated on top of the terrace, the closest to its edge. Also the gravity values along the profile are shown.

As mentioned, the actual profile measured does

not follow the terrace itself exactly, but is cut into it. In fact the 30 meter height difference observed at the terrace is crossed within 300 meters of levelling profile on the road. This also reflects in observed gravity values that continue to increase below the terrace until the distance of 300 meters from the edge [see the way $g(\text{obs})$ does not go straight up at the edge of the terrace as it would when measuring across a completely vertical edge]. Therefore the vertical shape of the terrace shown on graphs is only illustrative.

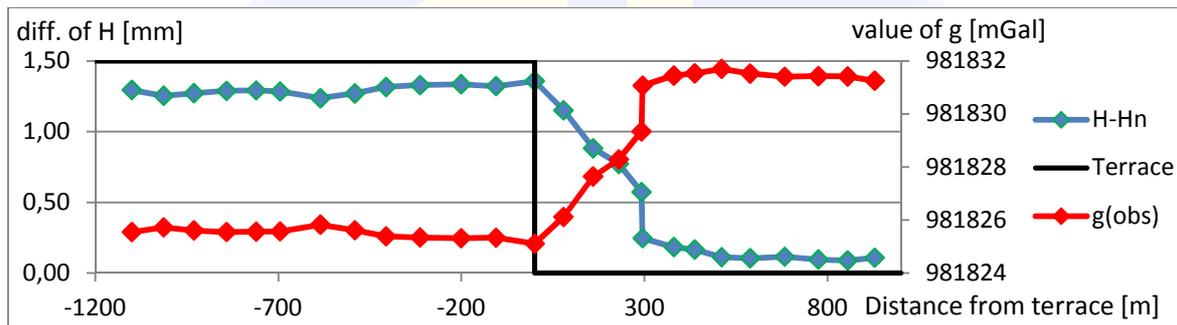


Figure 7. Cumulative gravimetric correction (depicted by the blue line) along the Tabasalu profile with the observed gravity values (depicted by the red line) and an illustrative shape of the terrace

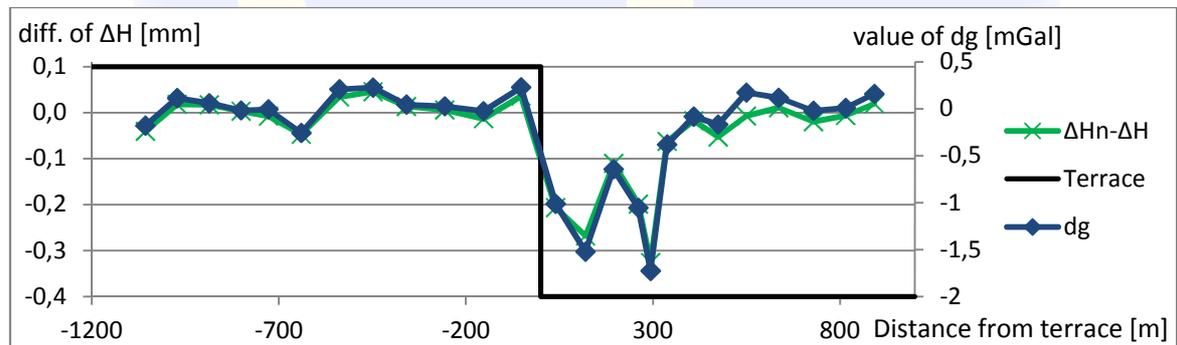


Figure 8. Gravimetric correction values (depicted by the green line) along the Tabasalu profile with the gravity change between two station points (depicted by the blue line) and an illustrative shape of the terrace

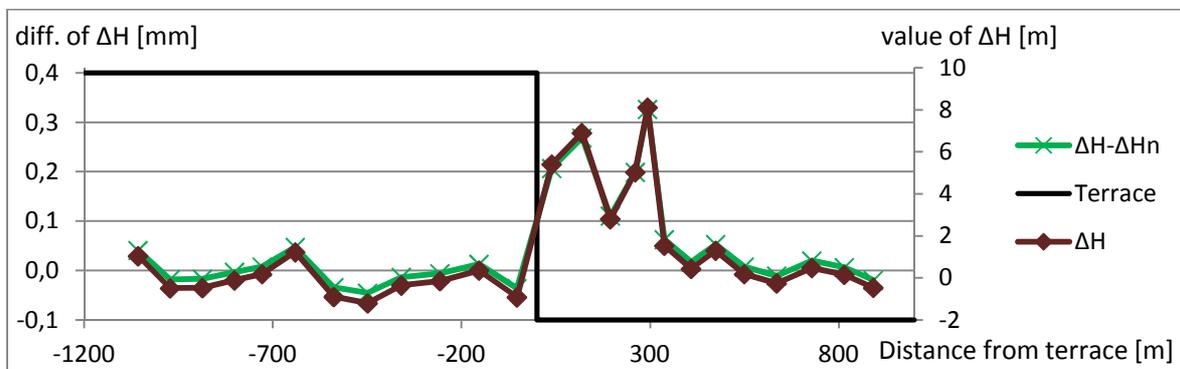


Figure 9. Gravimetric correction values (depicted by the green line) along the Tabasalu profile with the height change between two station points (depicted by the brown line) and an illustrative shape of the terrace

It becomes obvious that the gravimetric correction is the largest at the immediate neighbourhood of the terrace, where the height and gravity values change rapidly between levelling station points. This means that the 1.2 mm cumulative gravity correction is in fact distributed only within 300 meters, the immediate neighbourhood of the terrace.

To investigate further the correlation between the gravity change between two stations and the gravimetric correction, the following is done. The change in geometric heights ΔH between stations and the change in normal heights ΔH^n between stations are compared. The difference is plotted against the average distance of the two stations from the terrace (Figure 8). On the same graph, the change in gravity value between stations dg is shown.

A strong correlation is seen between the gravimetric correction and the gravity change. To a 1 mGal change in the gravity value corresponds roughly a 0.2 mm gravity correction.

A similar graph is plotted for the change of height ΔH against the gravimetric correction (Figure 9). Similarly, a strong correlation is found. To a change of height by 5 meters corresponds a 0.2 mm gravimetric correction value.

For numeric data and further information on the subject matters, please refer to the author's MSc thesis (Talvik, 2012).

Discussion

The gravimetric corrections calculated empirically have proved to be very significant – within 300 meters from the upper edge of the terrace (which corresponds to the actual length of the descent), the gravimetric correction can cumulate to 1.2 mm. This proves that the use of gravimetric corrections is vital in precise levelling data processing. The gravity data need to be known in levelling areas or collected alongside with the levelling fieldwork. Thus the quality of gravimetric data affects levelling accuracy directly.

The results are immense in terms of precise levelling. The existence of the gravimetric effect on precise levelling tends to be known among the geodesists but often neglected in practise. Therefore, the subject calls for further investigation to give more examples of the necessity of gravimetric corrections and to predict the magnitude of the gravimetric effect. In the current study the terrace reached a height of 30 meters. However the effect of higher terraces would be larger. A more extended discussion and numerical examples on the relation of height change and the gravity field change can be found in Talvik (2012). Using known information on the magnitude of the gravimetric correction, the effect of landforms with a different height can be predicted.

As mentioned, during the re-levelling of the Tallinn height network, a levelling section crossed the terrace on the same route. In their data processing, the gravimetric corrections were also used. It would be useful to compare the results of current research to the values obtained with the real levelling data. This would permit higher confidence in predicting the gravitational effect to precise levelling in similar areas elsewhere, also outside Estonia.

Summary

To evaluate the effect of gravity change along a levelling section, a field experiment on a terraced landform was conducted. Gravity data were acquired using a relative spring gravimeter; uncertainty of 0.07 mGal was achieved. Positioning was proceeded by using GPS technology; the uncertainty of achieved coordinates was likely not exceeding 0.15 m. Data collected were processed as if they had the accuracy of precise levelling.

Differences of height were converted into differences of geopotential number and later calculated into conventional normal height values. This process eliminates the effect of the Earth's gravity field from levelling results. By comparing the measured geometrical height differences with the gravimetrically corrected



ones, the magnitude of the gravimetric effect was found. The results indicate a strong correlation between both the height change and gravity change along the levelling profile and the gravimetric correction.

The change of gravity values along the levelling profile proved to have a significant effect on the levelling results – 1.2 mm in case of the current Tabasalu example. This result is the basis on which to continue investigating the magnitude of the gravimetric effect to levelling in more challenging regions.

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